Problem Set 1

1. *Show, by induction, the Bernoulli inequality:* $x > -1 \implies (1+x)^n \geq 1 + nx \;\; \forall n \in \mathbb{N}$
2. *Show, by contradiction, that the set of prime numbers is infinite.*
3. *Show the supremum of a set of real numbers is unique.*
4. *Let* A *and* B *be non-empty real-valued sets bounded above. Let* $C = \{a + b, a \in A, b \in B\}$*. Show* $\sup C = \sup A + \sup B$
5. *Given a real sequence* (a_j)*, define*

$$b_n = \sum_{j=1}^{n} a_j \qquad c_n = \sum_{j=1}^{n} |a_j|$$

   Show (b_m) *converges if* (c_m) *converges. Give an example of* (a_j) *to show the converse may not hold.*

6. *Show if* (x_m) *is a bounded and monotonic real sequence then* (x_m) *converges.*